NEAR-FIELD RADIATIVE HEAT TRANSFER BETWEEN GRAPHENE-COVERED ANISOTROPIC MAGNETO-DIELECTRIC HYPERBOLIC MATERIALS

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ABSTRACT

The near-field radiative heat transfer (NFRHT) between the anisotropic magneto-dielectric hyperbolic materials (AMDHMs) substrates covered by a monolayer of graphene is theoretically investigated on the basis of the fluctuational dissipation theorem through the scattering method. Unlike that the NFRHT flux between SiC nanowires can be enhanced by graphene coverage, graphene coverages can weakly impact on the NFRHT flux between the AMDHMs. This is mainly because that graphene plasmons cannot interact with the surface phonon polaritons (SPhPs) and hyperbolic modes supported by the AMDHMs in the low frequency region. Comparing to the NFRHT flux between graphene-covered AMDHMs on one side, the mismatch of reflection coefficients caused by the modification of graphene coverage on one side of the AMDHMs enables it to act like a thermal barrier layer in the heat transfer. Furthermore, tuning the geometry of the AMDHMs and the conductivity of graphene can effectively alter the hyperbolic modes and SPhPs for p and s polarizations which play dominant roles in the NFRHT. We found that choosing a suitable chemical potential and a larger relaxation time of graphene can to some extent enhance the NFRHT flux between the AMDHMs. This study could provide a guideline for studying the NFRHT involving hyperbolic metamaterials as well as graphene, and may be beneficial to further tune the NFRHT flux between the AMDHMs.

KEY WORDS: Radiative transfer; Tunneling; Metamaterials; Surface waves.

1. INTRODUCTION

When two thermally nonequilibrium objects are separated far away from each other, the radiative heat transfer between them is governed by the classical Stefan-Boltzmann law [1]. However, when the gap distance decreases to less than the characteristic wavelength of thermal radiation, the radiative heat transfer, namely near-field radiative heat transfer (NFRHT), cannot be predicted by the Stefan-Boltzmann law [2]. The NFRHT flux can even exceed the blackbody radiation limit by several orders of magnitude when surface wave modes, including surface plasmon polaritons (SPPs) and surface phonon polaritons (SPhPs), are excited [3]. This is because in the near-field regime the evanescent waves play dominant roles in energy transmission through photon tunneling rather than propagating waves [4]. So, NFRHT has attracted extensive attention due to the potential applications including but not limited to contactless thermal management [5, 6], thermal imaging [7, 8] and thermophotovoltaic (TPV) system [9].

A large body of theoretical and experimental investigations have demonstrated that except from surface modes featured with narrow resonance band, broadband resonance-free hyperbolic modes can enable the NFRHT to enhance in a broader frequency range [10, 11]. Hence, many studies on the NFRHT involving hyperbolic materials (HMs) have been carried out [12, 13]. Compared with the conventional uniaxial anisotropic HMs featured with a permittivity tensor \( \hat{\varepsilon} \), the anisotropic magneto-dielectric hyperbolic materials (AMDHMs) simultaneously have a permittivity tensor \( \hat{\varepsilon} \) and a permeability tensor \( \hat{\mu} \) which enable the AMDHMs to

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support hyperbolic modes and surface polaritons for p and s polarizations [14, 15]. Graphene has drawn enormous attention due to its exceptional optoelectric properties [16], and has been reported to utilize the NFRTH to improve the photoelectric conversion efficiency of near-field TPV system [17, 18]. Additionally, when doped silicon nanowires (NWs) are coated on graphene, the hybridization of SPPs of graphene and hyperbolic modes of doped silicon NWs enables the NFRHT to achieve 80% of a theoretical limit of hyperbolic materials [19]. So, a problem that what happens when the AMDHMs substrates are covered by a monolayer of graphene in the near-field regime arises. In this work, the NFRHT between the AMDHMs covered by graphene is theoretically investigated on the basis of the fluctuational dissipation theorem (FDT) through the scattering method. Our results indicate that graphene coverage weakly affects the NFRHT between AMDHMs, and the mechanisms are investigated by comparing the NFRHT flux between AMDHMs covered by graphene with SiC NWs covered by graphene. Additionally, considering that the AMDHMs and graphene are of excellent tunability, the effects of the geometry of AMDHMs and the conductivity of graphene on the spectral NFRHT are also examined.

2. THEORETICAL MODELING

Two semi-infinite AMDHMs substrates at different temperature $T_1$ and $T_2$ are covered by a monolayer of graphene on both sides and separated by vacuum with thickness $d$ as illustrated in Fig. 1. Based on the FDT using the scattering method, formulations of NFRHT flux between them can be divided into the contributions of propagating modes and evanescent modes as follows [2]

$$Q^{p,s}_{\text{prop}} = \frac{1}{\pi} \int_0^\infty n(\omega, T_1, T_2) d\omega \int_0^{\infty} d\beta \frac{1-|r_{p,s}^{01}|^2}{4} (1-|r_{p,s}^{02}|^2)$$

$$Q^{p,s}_{\text{evan}} = \frac{1}{\pi} \int_0^\infty n(\omega, T_1, T_2) d\omega \int_0^{\infty} d\beta \operatorname{Im}(r_{p,s}^{01}) \operatorname{Im}(r_{p,s}^{02}) e^{2ik_{zd}}$$

where $n(\omega, T_1, T_2) = \frac{\hbar \omega}{e^{\hbar\omega/k_BT_1} - 1} - \frac{\hbar \omega}{e^{\hbar\omega/k_BT_2} - 1}$ denotes the difference of the mean energy of the Planck oscillator per angular frequency at $T_1$ and $T_2$, $\omega$ is the angular frequency, $c$ is the speed of light, $\beta$ is the parallel component of the wave vector along the interface, and $k_{z0} = \sqrt{(\omega/c)^2 - \beta^2}$ is the z-component of wave vector.
in vacuum. Note that the last integral term after $\beta$ in Eqs. (1) and (2) is the transmission coefficient where $r_{0j}^{p,s}$ and $r_{1j}^{p,s}$ are the Fresnel reflection coefficients at the interface between AMDHMs substrates covered by a monolayer of graphene and vacuum for p and s polarizations. In order to obtain the reflection coefficients, we first need to define the surface conductivity of graphene, and the permittivity tensor and the permeability tensor of the AMDHMs. The surface conductivity of graphene is modelled as the sum of intraband (Drude) and interband contributions as follows [20]

\[ \sigma_{\text{intra}} = \frac{i}{\omega + \frac{\mu_e}{\tau_e}} \frac{2e^2k_BT_g}{\pi \hbar^2} \ln[2\cosh\left(\frac{\mu_e}{2k_BT_g}\right)] \]

(3)

\[ \sigma_{\text{inter}} = \frac{e^2}{4\hbar} \left[ G(h\omega) + \frac{4i\hbar\omega}{\pi} \int_0^{\infty} \frac{G(\xi) - G(h\omega)}{(h\omega)^2 - 4\xi^2} d\xi \right] \]

(4)

where $G(\xi) = \sinh\left(\frac{\xi}{k_BT_g}\right)/\cosh\left(\frac{\xi}{k_BT_g}\right)$, $T_g$ is temperature of graphene, $\varepsilon_s$ and $\mu_s$ respectively stand for the chemical potential and relaxation scattering time of graphene. AMDHMs exhibit magneto-dielectric anisotropy because they simultaneously and periodically have magnetic response and dielectric response, which originate from the uniaxial tensors $\hat{\varepsilon}$ and $\hat{\mu}$. The $\hat{\varepsilon}$ and $\hat{\mu}$ of the AMDHMs can be expressed as follows [15] of the AMDHMs can be expressed as follows [15]

\[ \hat{\varepsilon} = \begin{pmatrix} \varepsilon_O & 0 & 0 \\ 0 & \varepsilon_O & 0 \\ 0 & 0 & \varepsilon_E \end{pmatrix}, \quad \hat{\mu} = \begin{pmatrix} \mu_O & 0 & 0 \\ 0 & \mu_O & 0 \\ 0 & 0 & \mu_E \end{pmatrix} \]

(5)

in which the subscripts O and E represent the directions for the electric field perpendicular (ordinary) and along (extraordinary) the optical axis (z-axis in this work), respectively. In this study, SiC nanowires array is embedded into magnetic metamaterials to realize the magnetodielectric anisotropy. The characteristic size of the AMDHMs is far less than the characteristic wavelength of thermal radiation, so the effective medium theory can be utilized to describe the components of $\hat{\varepsilon}$ and $\hat{\mu}$ [14]:

\[ \varepsilon_O = \varepsilon_h \frac{\mu_h(1 - f) + \varepsilon_i(1 + f)}{\mu_h(1 + f) + \varepsilon_i(1 - f)}, \varepsilon_E = \varepsilon_h(1 - f) + \varepsilon_i f \]

(6)

\[ \mu_O = \mu_h \frac{\mu_h(1 - f) + \mu_i(1 + f)}{\mu_h(1 + f) + \mu_i(1 - f)}, \mu_E = \mu_h(1 - f) + \mu_i f \]

(7)

here, the subscript $h$ and $i$ refer to the host metamaterials and the inserted SiC NWs, and $f$ is the volume filling factor of SiC NWs. For the inserted SiC NWs, the permittivity of SiC can be obtained by Lorentz oscillator model, $\varepsilon(\omega) = \varepsilon_{\infty} (\omega_L^2 - \omega^2 - i\Gamma\omega)/(\omega_L^2 - \omega^2 - i\Gamma\omega)$, in which the longitudinal optical phonon frequency $\omega_L = 1.827 \times 10^{14}$ rad/s, the transverse optical phonon frequency $\omega_T = 1.495 \times 10^{14}$ rad/s and the damping factor $\Gamma = 0.9 \times 10^{12}$ rad/s [21]. For the host metamaterials, the permittivity and permeability can be given as $\varepsilon_h(\omega) = 1 - \omega_p^2/(\omega^2 + i\gamma_e \omega)$ and $\mu_h(\omega) = 1 - F \omega_c^2/(\omega^2 - \omega_c^2 + i\gamma_m \omega)$,where $\omega_p = 3.0 \times 10^{14}$ rad/s, $\omega_c = 1.0 \times 10^{14}$ rad/s, and $\gamma_e = \gamma_m = 1.0 \times 10^{12}$ rad/s [22]. $F$ denotes the volume filling factor of the interior cell of the host magnetic HMs. After describing the properties of graphene and AMDHMs, the Fresnel reflection coefficient for p and s polarizations can be calculated from

\[ r_{0j}^{p} = \frac{\varepsilon_O k_{0} - k_{j}^p + \sigma_g k_{0} k_{j}^p/\omega\varepsilon_0}{\varepsilon_O k_{0} + k_{j}^p + \sigma_g k_{0} k_{j}^p/\omega\varepsilon_0} \]

\[ r_{1j}^{p} = \frac{\varepsilon_O k_{0} - k_{j}^p + \sigma_g k_{0} k_{j}^p/\omega\varepsilon_0}{\varepsilon_O k_{0} + k_{j}^p + \sigma_g k_{0} k_{j}^p/\omega\varepsilon_0} \]

(8)
\[ k_{ij}^p = \frac{\mu_0 k_{ij}^0 - \sigma_{ij} \mu_0 \omega}{\mu_0 k_{ij}^0 + \sigma_{ij} \mu_0 \omega} \]  

\[ k_{ij}^s = \sqrt{\varepsilon_0 \mu_0 (\omega/c)^2 - (\varepsilon_0 / \varepsilon_E) \beta^2} \] 

where \( \varepsilon_0 \) and \( \mu_0 \) are the permittivity and permeability of vacuum, the subscripts 0 and \( j = 1 \) or 2 respectively denote the vacuum and the AMDHMs covered by graphene, so \( k_{ij}^p = \sqrt{\varepsilon_0 \mu_0 (\omega/c)^2 - (\varepsilon_0 / \varepsilon_E) \beta^2} \) and \( k_{ij}^s = \sqrt{\varepsilon_0 \mu_0 (\omega/c)^2 - (\mu_0 / \mu_E) \beta^2} \).

3. RESULTS AND DISCUSSION

![Graph](image-url)

**Fig. 2** Curve Plots of the real parts of the components of (a) permittivity tensor \( \varepsilon \) and (b) permeability tensor \( \mu \) of the AMDHMs with \( F = f = 0.4 \).

The dispersion relations of AMDHMs can be expressed as follows [23],

\[ \frac{k_x^2}{\varepsilon_E} + \frac{k_y^2}{\varepsilon_E} + \frac{k_z^2}{\varepsilon_O} = \mu_0 \left( \frac{\omega}{c} \right)^2 \text{ for } p \text{ polarization} \]  

\[ \frac{k_x^2}{\mu_E} + \frac{k_y^2}{\mu_E} + \frac{k_z^2}{\mu_O} = \varepsilon_0 \left( \frac{\omega}{c} \right)^2 \text{ for } s \text{ polarization} \]
here, \( k_x \), \( k_y \) and \( k_z \) respectively denote the \( x \), \( y \) and \( z \)-components of the wave vector. In order to obtain the dispersion relation with hyperbolic isofrequency surface, the signs of the components of each tensor should be opposite, namely \( \varepsilon_0 \varepsilon_k < 0 \) for \( p \) polarization or/and \( \mu_0 \mu_k < 0 \) for \( s \) polarization [24]. Figure 2 depicts the real parts of the components of permittivity tensor and permeability tensor with \( F = f = 0.4 \). The gray areas represent the hyperbolic dispersion relation with negative ordinary components, while the blue areas stand for that with positive extraordinary components. It is clearly seen that AMDHMs support hyperbolic modes for both polarizations which enable the NFRHT flux between two AMDHMs substrates to exhibit high values in these frequency regions as Fig. 3 shows. Hereinafter \( T_1 = 310 \) and \( T_2 = 290 \) K for AMDHMs, while \( \mu_g = 0.4 \) eV, \( T_g = 300 \) K, and \( \tau_g = 1 \times 10^{13} \) s for graphene as well as \( d = 100 \) nm unless otherwise specified.

The total NFRHT flux between two AMDHMs substrates without graphene coverage is 18.49 kW/m\(^2\) which is the sum of 12.66 kW/m\(^2\) for \( p \) polarization and 5.83 kW/m\(^2\) for \( s \) polarization. Compared to the case that the \( s \)-polarized NFRHT flux between two nonmagnetic substrates is far less than the \( p \)-polarized flux and can be neglected, the \( s \)-polarized NFRHT flux between two AMDHMs substrates is only a little less than half of the corresponding \( p \)-polarized flux. The hyperbolic regions of AMDHMs can be referred to the dispersion relations of the components of \( \hat{\varepsilon} \) and \( \hat{\mu} \) as shown in Fig. 2. These regions can also be identified by the frequency ranges with high NFRHT flux in Fig. 3. By zeroing the denominator of the Fresnel reflection coefficients in Eqs. (8) and (9) for \( p \) and \( s \) polarizations, as the following equations,

\[
\varepsilon_0 k_{z0} + k_{z1p} + \frac{\sigma_g k_{z0} k_{z1p}}{\omega \varepsilon_0} = 0 \quad \text{for} \ p \ \text{polarization} \tag{12}
\]

\[
\mu_0 k_{z0} + k_{z1s} + \sigma_g \mu_0 \omega = 0 \quad \text{for} \ s \ \text{polarization} \tag{13}
\]

one can obtain the expression of the resonance wavevectors, and then can search for the asymptotic parts of the dispersion relation which correspond to SPhPs resonance of AMDHMs by zeroing the denominator of the expression. When no graphene is covering, \( \sigma_g = 0 \). Then, we can obtain 0.58, 0.98 and 1.81 \( \times 10^{14} \) rad/s for \( p \) polarization and 1.04 \( \times 10^{14} \) rad/s which can be identified by several narrow peaks in Fig. 3. When the AMDHMs are covered by graphene, the nonzero term \( \sigma_g \) modifies the expression of the resonance.
wavevectors and lead to the blueshift of the resonance frequencies for p polarization but little change for s polarization. Furthermore, for p polarization, the hyperbolic regions move to higher frequencies except for the region from $1.13 \times 10^{14}$ rad/s to $1.49 \times 10^{14}$ rad/s. Adding graphene on AMDHMs can hardly change the location of the hyperbolic regions for s polarization as well as the resonance frequencies for s polarization. Furthermore, the epsilon-near-zero (ENZ) behavior enables the peak to appear at $2.03 \times 10^{14}$ rad/s, which originates from the enhanced light-matter interaction due to the topological transitions in hyperbolic metamaterials [25, 26]. The total NFRHT flux between AMDHMs covered by graphene decrease to 17.82 kW/m$^2$ which is mainly attributed to the supression of energy transmission for p polarization.

![Contour plots of transmission coefficients between (a) suspended graphene, (b) SiC NWs and (c) SiC](image)

**Fig. 4** Contour plots of transmission coefficients between (a) suspended graphene, (b) SiC NWs and (c) SiC
NWs covered by graphene versus the angular frequency and normalized wavevector for p polarization. The volume filling factor of SiC NWs $f = 0.4$.

In order to explore why the graphene coverage weakly affects on the NFRHT flux between AMDHMs, the cases of the NFRHT between SiC NWs with and without graphene coverage are discussed. The transmission coefficients between suspended graphene, SiC NWs array and graphene-covered SiC NWs array versus the angular frequency and normalized wavevector for p polarization are given in Fig. 4. In Fig. 4(a), two explicit bright bands denote the coupled SPPs in graphene which greatly contribute to the energy transmission between suspended graphene. The dispersion relations of two branches can be obtained by searching for the poles of the transmission coefficients [27]. The broad bright areas and the neighbouring narrowband peak in Fig. 4(b) demonstrate that SiC NWs support hyperbolic modes and SPhPs resonance for p polarization. When SiC NWs array is covered by graphene, the interaction between graphene plasmons, SiC SPhPs resonance modes and hyperbolic modes brings about two broad bright bands. It is precisely because the combination of such extraordinary propagating modes, the NFRHT flux between SiC NWs with graphene coverage can reach 20.29 kW/m$^2$ which is seven times larger than that between SiC NWs without graphene coverage. However, the contribution of the coupled plasmons in graphene vanishes which leads to the weak effects on the NFRHT flux when graphene is coated on the AMDHMs as Fig. 5 shows. In Fig. 5(b), there exist no bright areas like in Fig. 4(c) in the low frequency region, such as $\omega = 0.3 \times 10^{14}$ rad/s. In order to explain this phenomenon, we calculate the numerator and denominator of the transmission coefficients for SiC NWs covered by graphene and AMDHMs covered by graphene. The numerator of the transmission coefficient is nonzero when the denominator equals zero for the former configuration, so the transmission coefficient exhibits a pole which enables the NFEHT flux to greatly increase. Whereas for the latter configuration, the transmission coefficient exhibits no extreme value because the denominator and the numerator of the transmission coefficient simultaneously equal zero. Hence, graphene plasmons cannot interact with the SPhPs and hyperbolic modes supported by AMDHMs in the low frequency region which leads to the weak impacts on the NFRHT flux. However, graphene plasmons in the high frequency region still exist as shown in Fig. 5(b) which to some extent contribute to the NFRHT flux and can also be demonstrated by the peak around $3 \times 10^{14}$ rad/s in Fig. 3.

![Fig. 5 Contour plots of transmission coefficients between (a) AMDHMs substrates and (b) AMDHMs substrates covered by graphene versus the angular frequency and normalized wavevector for p polarization. The volume filling factor of SiC NWs $f = 0.4$, and the interior cells in the host metamaterials $F = 0.4$.](image-url)

The above discussion is based on the AMDHMs substrates covered by graphene on both sides, and Fig. 6 shows the comparison of the spectral NFRHT flux between the AMDHMs substrates covered by graphene on one side and both sides for p and s polarizations. Note that graphene can be coated on the surface with higher temperature or on the lower one, and the NFRHT flux for both cases is identical with each other. The total NFRHT flux with graphene coverage on one side and both sides are 14.32 kW/m$^2$ and 17.82 kW/m$^2$ which are inferior to that without graphene coverage. It demonstrates the graphene coverage on one side behaves like a
thermal barrier layer to suppress the energy transmission between AMDHMs. When another monolayer of graphene is coated on the other side, the modification of reflection coefficient enables the hyperbolic regions for p polarization to broaden so that the total flux increases. For s polarization, the existence of graphene coverage hardly alters the NFRHT flux between the AMDHMs except in the low frequency region as shown in Figs. (3) and (6), and this is because the $\sigma g \mu_0 \omega$ is orders of magnitude less than the wavevector.

**Fig. 6** Spectral NFRHT flux between AMDHMs covered by graphene on one side and both sides for p and s polarizations with $f = F = 0.4$.

In consideration of the tunability of the AMDHMs, we investigate the effects of variant filling factors of SiC NWs $f$ and interior cell of the host metamaterials $F$ on the spectral NFRHT flux for p and s polarizations at $d = 100$ nm, as shown in Fig. 7. The change of $f$ can explicitly affect the p- and s-polarized NFRHT flux by altering the hyperbolic modes and SPhPs resonance. For p polarization in Fig. 7(a), it can be seen that hyperbolic regions with $\varepsilon_0 > 0$ and $\varepsilon_E < 0$ move to lower frequencies while those with $\varepsilon_0 < 0$ and $\varepsilon_E > 0$ hardly move when $f$ increases. Not only that, a larger $f$ can also enlarge the spectral NFRHT flux due to the hyperbolic modes with $\varepsilon_0 > 0$ and $\varepsilon_E < 0$ but reduce those with $\varepsilon_0 < 0$ and $\varepsilon_E > 0$. Additionally, the peaks due to SPhPs shift to lower frequencies with increasing $f$. For s polarization in Fig. 7(b), SPhPs resonance peak moves to a lower frequency while the hyperbolic regions move to higher frequencies with the increase of $f$. Note that with $f = 0.2$, a less pronounced peak due to the ENZ behavior at $9.65 \times 10^{14}$ rad/s appears which does not occur with $f = 0.4$ and 0.6. The total NFRHT flux with $f = 0.2$ for s polarization is 9.41 kW/m$^2$ and then decreases to 3.51 kW/m$^2$ with $f = 0.6$, while that for p polarization declines little from 11.61 kW/m$^2$ to 10.24 kW/m$^2$. Figures. 7(c) and (d) illustrate the effects of variant $F$ on the spectral NFRHT flux for p and s polarizations. Obviously, except for the regions near $1.5 \times 10^{14}$ rad/s and larger than $3.5 \times 10^{14}$ rad/s, variant $F$ has little effect on the spectral NFRHT flux for p polarization. However, for s polarization, a larger $F$ can heighten the peak due to SPhPs resonance and broaden the hyperbolic regions so that the total NFRHT flux with $F = 0.6$ reach 9.78 kW/m$^2$ which is approximately 4 times larger than that with $F = 0.2$. The peaks due to ENZ behavior almost overlap with variant $F$. The effects of variative $f$ and $F$ on the NFRHT flux are different which mainly originates from that $f$ simultaneously affects the permittivity and the permeability of the AMDHMs while $F$ just influences the permeability.
Fig. 7  Spectral NFRHT flux between AMDHMs covered by graphene with variant filling factors of the inserted SiC NWs $f$ for (a) $p$ and for (b) $s$ polarizations, and filling factors of interior cells of the host metamaterials $F$ for (c) $p$ and for (d) $s$ polarizations.
Graphene is gapless semimetal with high tunability, for instance the chemical potential can be modulated by chemical doping and external electromagnetic fields [16]. The effects of tuning the chemical potential \( \mu_g \) and relaxation time \( \tau_g \) on the spectral NFRHT flux for p and s polarizations at \( d = 100 \) nm are considered as shown in Fig. 8. For p polarization, a higher \( \mu_g \) leads to a less magnitude of the spectral NFRHT flux due to SPhPs as well as hyperbolic modes with \( \varepsilon_0 > 0 \) and \( \varepsilon_E < 0 \) which enables the p-polarized NFRHT flux to decrease from 12.25 kW/m\(^2\) to 8.95 kW/m\(^2\), and move the locations of them to higher frequencies, while the locations of hyperbolic modes with \( \varepsilon_0 < 0 \) and \( \varepsilon_E > 0 \) hardly change. In comparison, a higher \( \tau_g \) slightly increases the p-polarized NFRHT flux from 10.76 kW/m\(^2\) to 12.19 kW/m\(^2\). Evidently, both of \( \mu_g \) and \( \tau_g \) affect little on the spectral s-polarized NFRHT flux. Considering there exists a maximum value of the total NFRHT flux in terms of the chemical potential when \( \mu_g = 0.2 \) eV. So choosing the optimal \( \mu_g \) and a larger \( \tau_g \) will further enhance the energy transmission between the AMDHMs covered by graphene. For instance, the total NFRHT flux is 19.14 kW/m2 which is 5% larger than that without graphene coverage when \( \mu_g = 0.2 \) eV and \( \tau_g = 50 \times 10^{-13} \) s.

**4. CONCLUSION**

In this study, we have theoretically investigated the NFRHT between two AMDHMs substrates covered by graphene. It was found that the graphene coverage cannot greatly impact on the NFRHT flux between AMDHMs. Via referring to the case of graphene-covered SiC NWs, we can demonstrate that this is mainly because graphene plasmons cannot interact with the SPhPs and hyperbolic modes supported by AMDHMs in the low frequency region which contributes a lot to the NFRHT flux in the case of graphene-covered SiC NWs. Compared with the NFRHT flux for graphene coverage on both sides, graphene coverage on one side acts like a thermal barrier layer due to the mismatch of reflection coefficients due to the existence of graphene coverage. Tuning the volume filling factor of SiC NWs can greatly change the heat flux between AMDHMs covered by graphene for both polarizations, while that of the interior cells of the host metamaterials \( F \) almost only influences the s-polarized NFRHT flux. Choosing the optimal \( \mu_g \) and a larger \( \tau_g \) based on the tunability of graphene can slightly enhance the NFRHT flux between AMDHMs. This study could provide a guideline for studying the NFRHT involving hyperbolic metamaterials as well as graphene, and has some benefits to explore more approaches to further tune the NFRHT flux between AMDHMs.

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